

On the small-ball approach in non-parametric regression

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In a series of papers, Mendelson and Lecué & Mendelson recently developed the so-called small-ball approach, that allows to deal with efficiency of (regularized) Empirical Risk Minimizers without concentration arguments. Looking at the small-ball property in the context of non-parametric regression, we show however that the bounds obtained by Lecué and Mendelson systematically lead to suboptimal rates for a variety of functional bases, such as piecewise polynomials, wavelets or the Fourier basis. A refinement of the method is thus needed and particularizing to the case of projection estimators on the Fourier basis, we show that mild regularity assumptions on the regression function allow to localize the small-ball argument in such a way that optimal bounds are recovered.